Upon substitution of (40) into the governing equation (23), we see that (23) will be satisfied identically provided

$$\sum_{j=1}^{\infty} \frac{C_j}{A_j} J_o(\lambda_j r) \left(\frac{1}{\theta_1} - \upsilon \lambda_j^2\right) = \frac{P_g}{\rho_L}$$
(41)

If we prescribe that

$$A_{j} = \left(\frac{1}{\theta_{1}} - \upsilon \lambda_{j}^{2}\right)$$
(42)

then (41) becomes

$$\sum_{j=1}^{\infty} C_{j} J_{0} (\lambda_{j} r) = \frac{P_{g}}{\rho L}$$
(43)

According to the Fourier-Bessel expansion, the constants C j are given as

$$C_{j} = \frac{\frac{P_{g}}{\rho_{L}} \int_{0}^{R_{o}} r J_{o} (\lambda_{j}r) dr}{\frac{1}{2}R_{o}^{2} [J_{o}^{2} (\lambda_{j}R_{o}) + J_{1}^{2} (\lambda_{j}R_{o})]}$$
(44)

From equation (40) we see that the boundary condition (26) will be satisfied providing we define  $\lambda_j$  as the positive roots of the equation

$$J_{o}(\lambda_{j}R_{o}) = 0$$
(45)

Combining equation (44) and (45), we have

$$C_{j} = \frac{2P_{g}}{\rho_{LR_{o}\lambda_{j}}} \left[\frac{1}{J_{1}(\lambda_{j}R_{o})}\right]$$
(46)

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Inserting (42) and (46) into (40), we obtain

$$w(\mathbf{r},t) = \frac{2P_g}{PLR_o} \sum_{j=1}^{\infty} \frac{J_o(\lambda_j \mathbf{r})}{\lambda_j J_1(\lambda_j R_o)} \left[ \frac{1}{\frac{1}{\theta_1} - \upsilon \lambda_j^2} \right]$$

$$\cdot \left[ e^{-\lambda_j^2 \upsilon t} - e^{-t\theta_1} \right]$$
(47)

An inspection of equation (47) shows that conditions (24), (25), and (26) are satisfied. Thus equation (47) is the formal solution of the subject boundary value problem.

The volume rate of fluid Q passing through the tube in time t can be determined as follows:

$$Q = \int_{0}^{A} w(r,t) da$$
 (48)

where A is the cross-sectional area of the bore of the knockoff tube. Substituting equation (47), we have

$$Q = \frac{4\pi P_g}{PLR_o} \sum_{j=1}^{\infty} \left[ \frac{1}{\frac{1}{\theta_1} - \upsilon \lambda_j^2} \right] \left[ e^{-\lambda_j^2 \upsilon t} - e^{-t/\theta_j} \right]$$

$$\cdot \left[ \frac{1}{\lambda_j J_1(\lambda_j R_o)} \right] \int_0^{R_o} r J_o(\lambda_j r) dr$$
(49)

from which

$$Q = \frac{4\pi P_g}{PL} \sum_{j=1}^{\infty} \left[ \frac{1}{\frac{1}{\theta_1} - \upsilon \lambda_j^2} \right] \left[ e^{-\lambda_j^2 \upsilon t} - e^{-t/\theta_1} \right] \left( \frac{1}{\lambda_j^2} \right)$$
(50)